TANGENT MODULUS OF PILES
DETERMINED FROM
STRAIN DATA

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TANGENT MODULUS OF PILES DETERMINED FROM STRAIN DATA

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ABSTRACT: When evaluating measurements of strain or telltale shortening of a pile subjected to loading test, precise knowledge of the "elastic" modulus is necessary. By means of plotting and analyzing the increment of load (or stress) over the increment of strain versus the strain, i. e., the tangent or chord modulus of the stress-strain curve, the modulus can be determined. The paper provides the mathematical background to the analysis and presents examples of the method.

PRINCIPLES OF STRESS-STRAIN ANALYSIS

Often in a static pile loading test, the pile is instrumented with strain gages or telltales. The gages serve to determine the axial strain induced in the pile by the applied load and the strain data are used to evaluate the load distribution in the pile. The evaluation requires two important aspects: one, knowledge of the "elastic" modulus of the pile cross section and, two, measurements of high accuracy.

Fig. 1A shows a typical stress-strain diagram of data from an instrumented loading test. The line with "data" points that is curved near the origin and becomes linear toward higher strains, the upper line, indicates "measured" data. The line which is straight from the origin, the lower line, is the theoretical elastic line for a column with equal properties to that of the pile. The difference between the lines is, of course, due to shaft resistance acting on the pile in the loading test.

All shaft resistance has been overcome in the test, when the "measured" curve becomes parallel to the "theoretical". When evaluating the results from a loading test, finding this point is desirable, although, in practice, its location is often difficult to determine.

However, by plotting the tangent modulus of the "measured" curve", the point becomes easily discernible. The tangent modulus is the slope of the curve and it is plotted as the increment of load divided by the increment of strain plotted against the strain. The tangent modulus plot of the stress-strain lines is shown in Fig. 1B.

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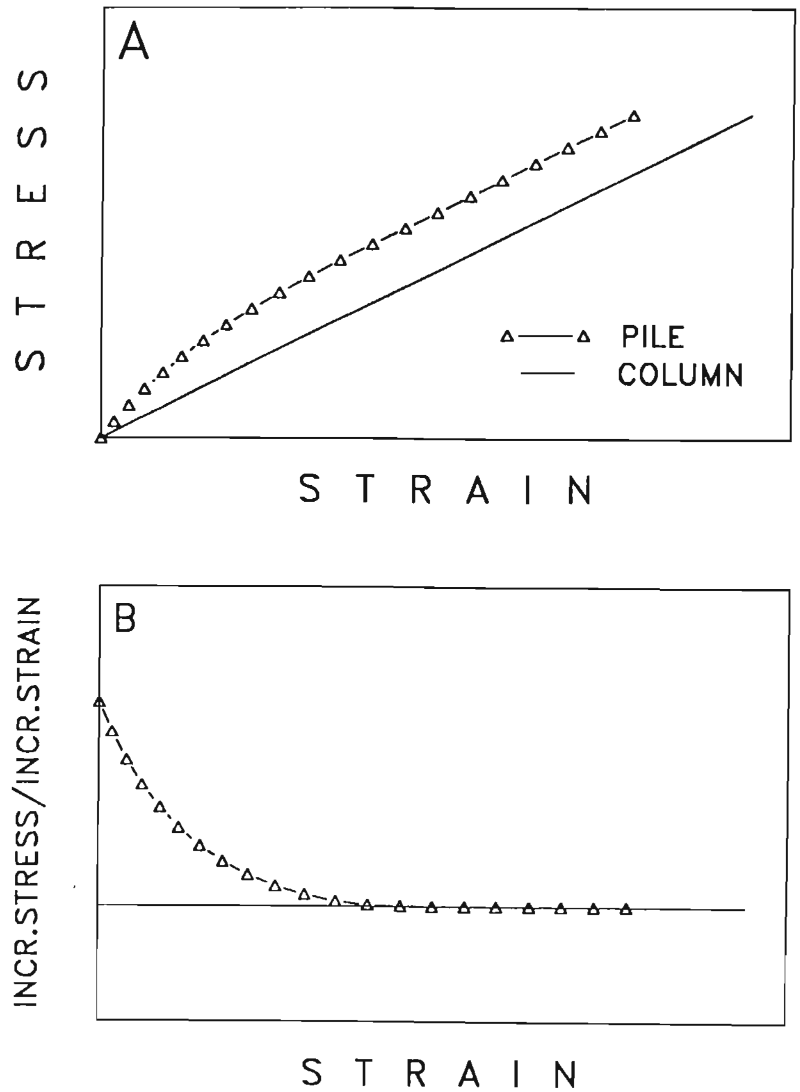

Figure 1. Typical data from an instrumented static pile loading test on a pile with a constant modulus.
A. Stress-strain diagram of the pile head (upper curve) and of the corresponding free standing column.
B. Plot of tangent modulus against strain.

As shown in Fig. 1B, the tangent modulus, or, more correctly termed, the chord modulus initially reduces with increasing strain to become constant at a certain amount of strain. This occurs when all the shaft resistance has been overcome and the constant value is equal to the pile modulus.

Often, the exact modulus of the test pile is not known. Then, the tangent modulus plot becomes a valuable aid in determining the modulus, which then is used in the calculations to determine the distribution of the load in the pile.

ACTUAL TEST RESULTS

Fig. 2 presents actual test results from a loading test to 2,670 KN (300 tons) on an H-pile (310HP93; 12HP63) installed at Jones Island, Milwaukee, Wisconsin. The figure shows the load-movement diagram of the pile head, and the movement of the pile toe and the compression of the full length of the pile (really the load applied to the pile head plotted against the toe movement and the compression).

The pile was equipped with two telltales, one upper to a depth of 39.3 m (128.8 feet) and one to the toe of the pile at a depth of 48.3 m (158.5 feet). The telltales were inserted into a standard one-inch pipe attached to the pile by occasional welds. The cross sectional area of the pile was measured (by weighing a short section) to 122.0 cm^2 (18.9 in^2), which is 3 % larger than the nominal area. The nominal area of the guide pipes was 6.5 cm^2 (1.0 in^2). Thus, the total cross sectional area not including the welds was 128.5 cm^2 (19.9 in^2).

A load cell was used to determine load. The reading precision of the dial gages for movement and shortening due to compression for the applied load was in a gradation of 0.025 mm (0.001 inch).

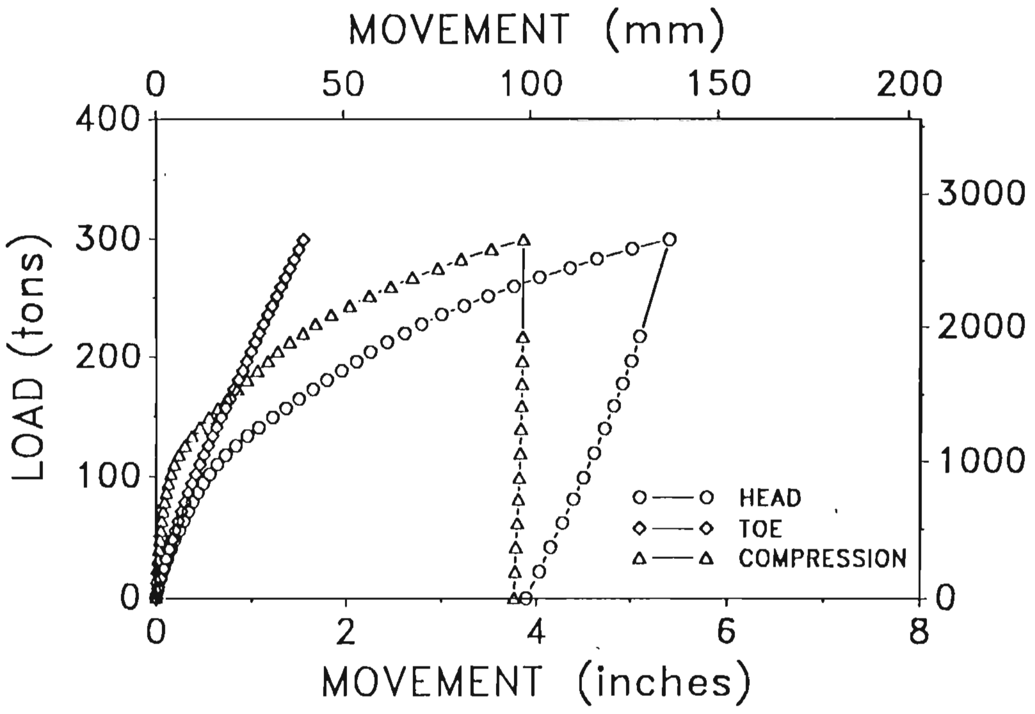

Figure 2. Actual results from a static loading test on an H-pile showing movement of the pile head and the pile toe, and and measured axial compression of the pile plotted against the applied load.

Fig. 3A shows the applied load at the pile head plotted against measured strain (i. e., shortening divided by telltale length) for the upper and lower telltales and for the difference between the telltales, i. e., the strain along the bottom 9.0 m (29.7 feet) of the pile.

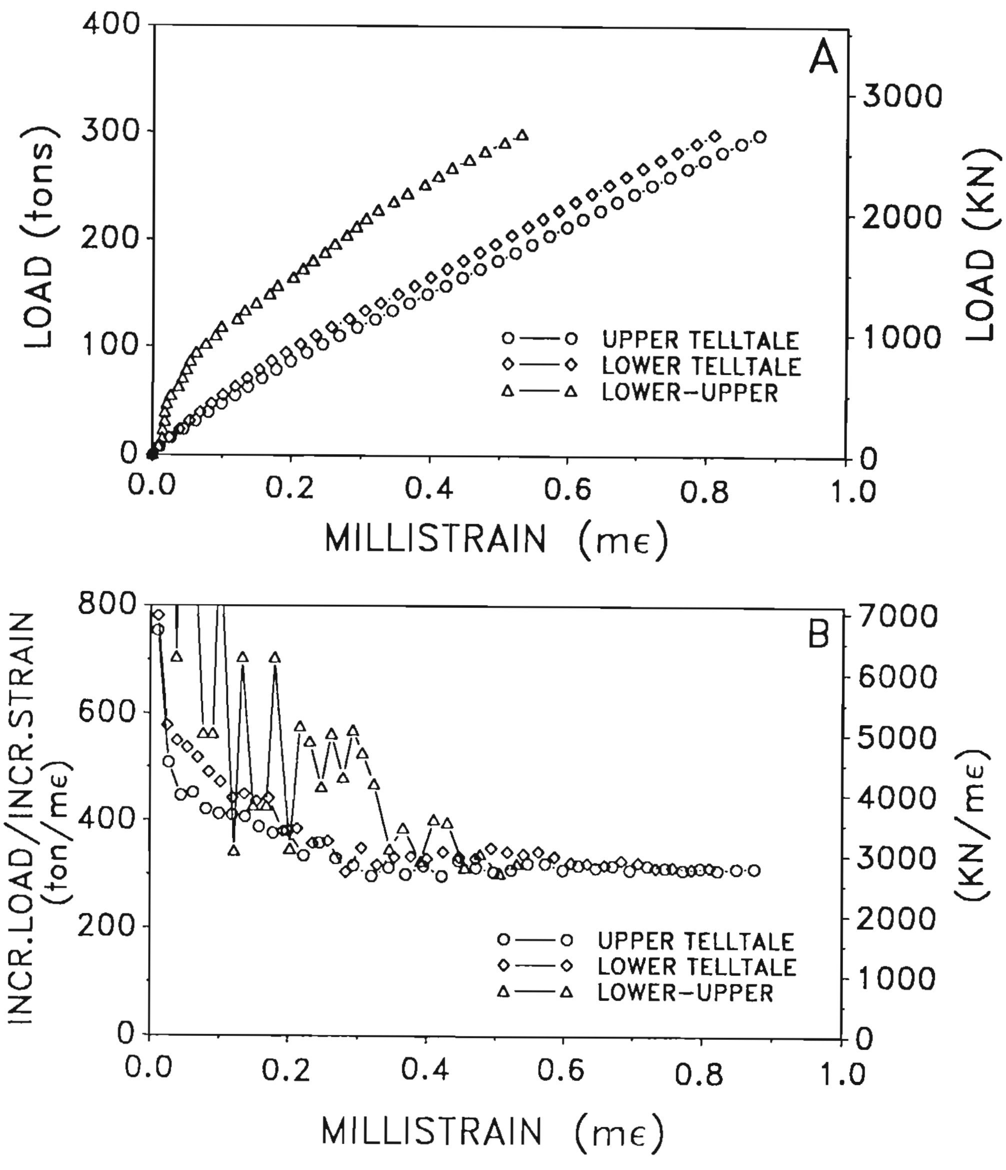

Figure 3. Load-strain and modulus diagrams for the pile data shown in Fig. 2.

A. Load-strain diagram for two telltale lengths and for the difference between the two telltale lengths.

B. Tangent modulus diagram for the two telltale lengths and for the difference between the two telltale lengths.

It is very difficult to obtain anything quantitative from the diagram in Fig. 3A. However, when studying the diagram in Fig. 3B showing the tangent modulus plot, it can easily be determined that the curve for the upper telltale indicates that a constant modulus (a horizontal, straight line portion) develops at a value of 0.3 millistrain, which occurred when the applied load was 1,070 KN (120 tons). For the lower telltale, a constant modulus is indicated for a strain of 0.8 millistrain occurring when the applied load was 2,450 KN (275 tons). Finally, the curve for the telltale difference (bottom portion of the pile) indicates a constant modulus at a strain of 0.5 millistrain at the applied load of 2,490 KN (280 tons).

The analysis of the tangent moduli for a range of applied load of 2,518 KN to 2,670 KN (283 to 300 tons) indicates a modulus for the upper, lower, and bottom portion telltale lengths, of 2.776, 2.785, and 2.847 MN/strain (312, 313, and 320 ton/millistrain). The agreement between the upper and lower telltale values is excellent. It is not surprising that the lower portion value is slightly off as any inaccuracy in the telltale readings would be exaggerated when taking the difference of them.

Thus, the evaluation indicates that the tangent modulus of the pile cross section is equal to 2.78 MN/strain (312 ton/millistrain). By inserting this value into the conventional relation LOAD = AREA times MODULUS times STRAIN with the cross sectional area equal to 128.5 cm^2 (19.9 in^2), an "elastic" modulus of 214 GPa (31,000 ksi) is obtained. Inserting the usual modulus value of 207 GPa (30,000 ksi), an area of 134.2 cm^2 (20.8 in^2) is calculated. Obviously, in the continued evaluation of the data, one has to live with an inaccuracy of the modulus, or actual cross sectional area, of about 4 % due to the uncertainty in the calculations. The Author finds this more than acceptable for engineering purposes.

The analysis becomes a little bit more difficult when evaluating strain data from other than steel piles, i. e., concrete piles or concrete-filled pipe piles. Contrary to common belief, a concrete column does not exhibit a linear stress-strain relation when loaded. That is, the Young's modulus of concrete reduces with the applied load. Fig. 4A illustrates an assumed stress-strain curve of a column (lower line). It has been assumed that the line is a second degree curve and that the final slope of the line is 30 % of the initial slope. This reduction of the slope, i. e., the modulus, is extreme, and has been chosen for reasons of instruction clarity. (An example of an actual case will be given later).

The upper curve in Fig. 4A, the line with the data points, shows the same column taken as a pile subjected to shaft resistance. As in the case of the pile with the constant modulus illustrated in Fig. 1, as soon as all the shaft resistance has been overcome, the two lines are parallel. Due to the curving of the lines, it is very difficult to tell when this occurs, however.

In Fig. 4B, the tangent modulus of the column line is plotted against the strain (solid line). Because the stress-strain relation for the column has been assumed to follow a second degree equation, the tangent modulus is a straight line, and, as the modulus is not constant but reducing, the line slopes downward with increasing strain. The line with the "data" points is the tangent modulus line for the pile. It becomes parallel with that of the column after the shaft resistance has been overcome. As shown, it plots slightly below the column line. Extrapolating the pile modulus line to the y-axis and integrating it, would result in a "restored column curve" located marginally below the true column stress-strain curve in Fig. 4A.

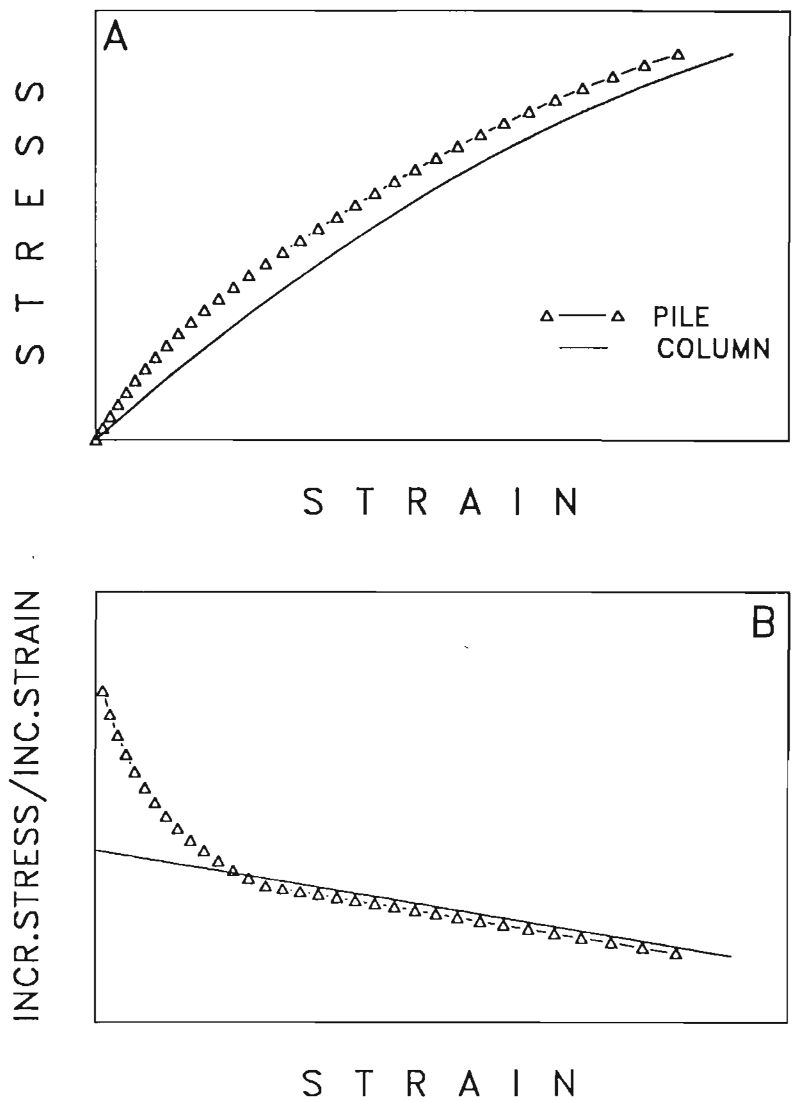

Figure 4. Typical data from an instrumented static pile loading test on a concrete pile with a modulus reducing with increasing stress.
A. Stress-strain diagram of the pile head (uppercurve) and of the corresponding free standing column.
B. Plot of tangent modulus against strain.

MATHEMATICAL RELATIONS

Mathematically, the lines and curves are expressed, as follows:

The equation for the tangent modulus line is:

$$E_t = d\sigma/d\epsilon = A\,\epsilon + B \quad (1)$$

where E_t = the tangent modulus, σ = the applied stress, ϵ = the induced strain, A = the slope of the tangent modulus line, and B = the Y-intercept (initial tangent modulus).

Integrating the tangent modulus line results in the following equation for the column line:

$$\sigma = \tfrac{1}{2}A\,\epsilon^2 + B\,\epsilon \quad (2)$$

And the stress in the pile for an induced strain:

$$\sigma = E_s\,\epsilon \quad (3)$$

where E_s = the secant modulus and

$$E_s = \tfrac{1}{2}A\,\epsilon + B \quad (4)$$

EXAMPLE FROM A PILE WITH A NON-CONSTANT MODULUS

The tangent modulus method of evaluation applied to piles of non-constant "elastic" modulus is illustrated by the results from a static loading test on a precast, prestressed concrete pile. The pile is a 62.8 m (206.0 feet) long 0.42 m (16.5 inch) octagonal pile installed for the Keehi Interchange, Honolulu, Hawaii, and equipped with eleven telltales to measure compression during the loading test. The applied load at the pile head is measured by means of a load cell and the dial gages for movement and compression had a gradation of 0.0025 mm (0.0001 inch). The test was carried to an applied load of 4540 KN (510 tons) at which load the pile broke. The pile toe movement at the maximum load was 6.6 mm (0.26 inch). Bearing failure was not reached.

Data from two telltales have been chosen for the illustration: Telltale 7 at a depth in the pile of 38.6 m (126.64 feet) and Telltale 9 at a depth of 50.2 m (164.62 feet), where the maximum movements measured for the telltale points were 11.3 mm (0.44 inch) and 24.9 mm (0.98 inch), respectively. These telltales were chosen for reasons of ensuring that all or most of the shaft resistance over the telltale lengths had been overcome at the maximum load, which is not the case for the lowest telltale lengths.

Fig. 5A shows the applied loads plotted against the measured strains over the two telltale lengths and over the difference between the two telltales. It is obvious that the lines are curved.

The question when seeing such curves is "are they curved because the shaft resistance is not yet fully overcome, or because the modulus is reducing with increasing load, or both"?

The answer to the question is given in Fig. 5B, showing the tangent modulus plot of the data. Indeed, the tangent modulus lines are becoming straight at larger strains, which suggests a second degree curve for the stress-strain relation.

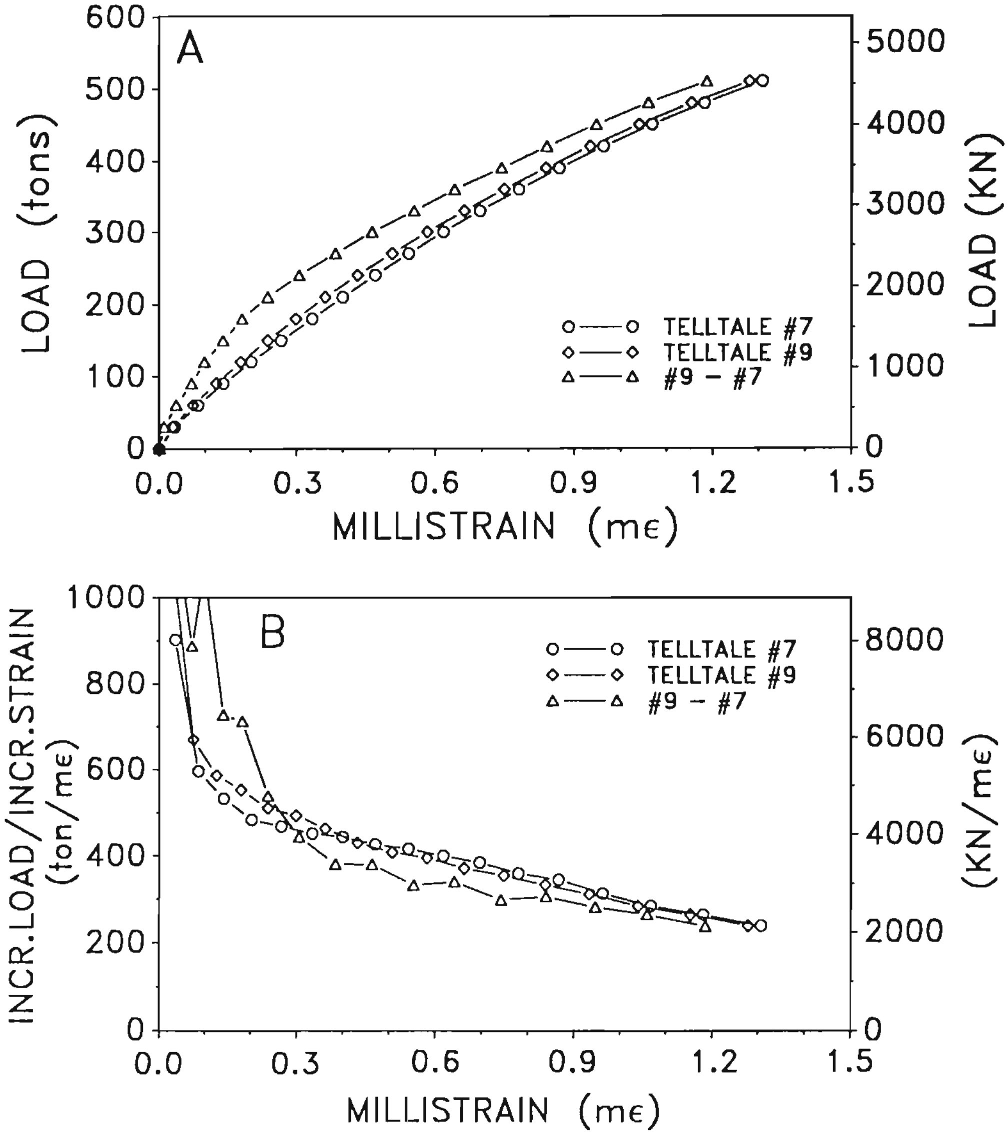

Figure 5. Load-strain and modulus diagrams from a static loading test on a prestressed concrete pile.
A. Load-strain diagram for two telltale lengths and for the difference between the two telltale lengths.
B. Tangent modulus diagram for the two telltale lengths and for the difference between the two lengths.

Telltale 7 indicates a straight line at a strain of 0.2 millistrain, induced at an applied load of about 1070 KN (120 tons). Telltale 9 indicates the same at a strain of 0.4 millistrain, induced at an applied load of about 2000 KN (225 tons). The line for the bottom portion indicates the straight line relation at a strain of about 0.8 millistrain, induced at an applied load of about 3560 KN (400 tons).

The tangent modulus lines are used to evaluate at what applied load the shaft resistance along the pile was fully mobilized. As to determining the load distribution for a particular applied load, knowledge is required of the secant "elastic" modulus for the load.

Linear regression of the data points making up the straight portion of the three lines may be used to provide the equation of the modulus lines, i. e., to determine the constants A and B in Eq. 1. Regression of the modulus line for Telltale 7 results in that the constants A and B are equal to -2.14 KN/millistrain (-0.240 ton/millistrain) and 4,877 KN/millistrain (548.2 ton/millistrain), respectively, with a linear regression correlation coefficient of 0.9980.

Applying Eqs. 1 through 4, results in the following values of initial and final tangent moduli, and final secant modulus:

Initial E_t = 37.8 GPa (5,480 ksi)

Final E_t = 16.2 GPa (2,340 ksi)

and E_s = 27.0 GPa (3,900 ksi)

Inserting the values of A and B into Eq. 2 gives the average load in the pile over the length of a telltale as a function of induced strain:

$$Q = -1.07\,\epsilon^2 + 4880\,\epsilon \quad \text{KN} \qquad (5a)$$

$$(Q = -0.12\,\epsilon^2 + 548\,\epsilon \quad \text{tons}) \qquad (5b)$$

Naturally, the tangent modulus method is not restricted to the analysis of telltale data. In fact, a considerable improvement of the accuracy of the load determination is obtained by using strain gages directly in lieu of telltales.

One of the most immediately noticed benefits of the tangent modulus method is that inaccuracies in the data become readily apparent. For instance, Fig. 6A shows the results from testing a 22 m (71 feet) long pipe pile with a diameter of 273 mm (10.75 inches) installed in Ottawa, Ontario. The pile was equipped with one telltale to the toe of the pile. The applied load was "controlled" only by means of the jack pressure (a highly unreliable way of measuring the load, but, unfortunately, very common in the engineering practice). The gradation of the dial gages was in 0.025 mm (0.001 inch).

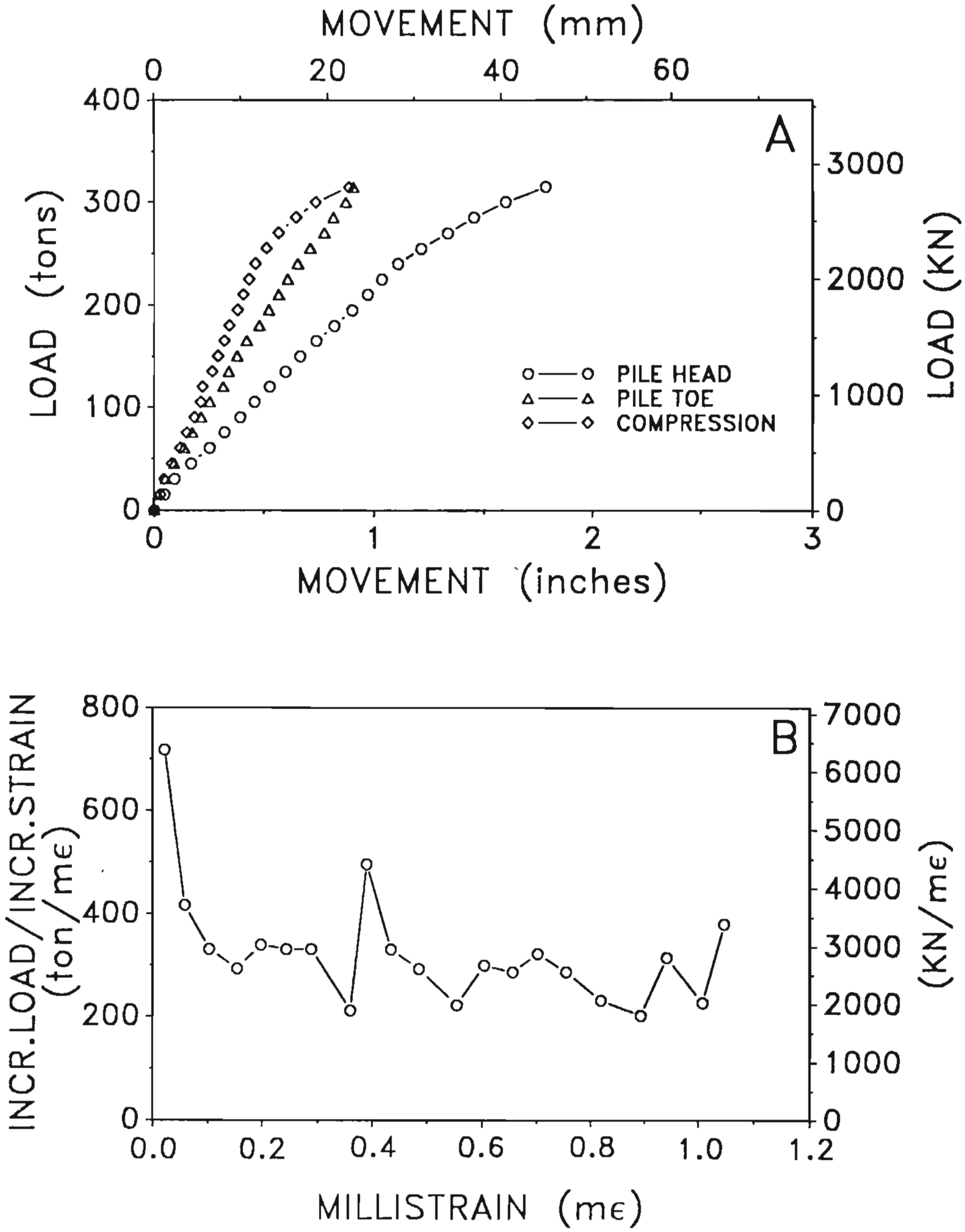

Figure 6. Actual results from a static loading test on a pipe-pile.
A. Movement of the pile head and the pile toe, and measured compression of the pile plotted against the applied load.
B. Tangent modulus diagram for the compression of the full pile length.

Fig. 6A does not indicate any inaccuracy in the data. However, the tangent modulus plot shown in Fig. 6B exhibits an erratic plot that would be very difficult to evaluate with an acceptable accuracy and no attempt to evaluation was or will be made.

Strain measurements, using telltale data or strain gages directly, can be evaluated as to accuracy, at what applied load the shaft resistance is fully mobilized, and as to what value of the secant modulus to use for determining the load distribution in the pile in the following analysis (which should consider factors such as the residual strain in the pile, as well as variation between individual gages).

To have any reasonable chance of obtaining data suitable for analysis — any analysis, in fact — the loading test must include load measurements by means of a reliable load cell and the strain data be obtained with an accuracy being about ten times greater than the one usually applied in engineering practice. As to telltale data, dial gages or LVDTs which give a precision of 0.0025 mm (0.0001 inch) are preferred over the commonly used "standard" gages having a gradation of 0.025 mm (0.001 inch).

CONCLUSIONS

When evaluating measurements of strain or telltale shortening of a pile subjected to loading test, precise knowledge of the "elastic" modulus is necessary before the induced load can be calculated from the measured strain or shortening. By means of plotting and analyzing the increment of load (or stress) over the increment of strain versus the strain, i. e., the tangent or chord modulus of the stress-strain curve, the modulus, E, can be determined. For steel piles, the Young modulus of the material is of course known, but the exact area is rarely known. Then, the tangent modulus approach assists in determining the AE-value of the pile cross section. For concrete-filled pipe piles and precast concrete piles, which may not only have an unknown modulus and area, but also a modulus that varies with the level of stress in the pile during the test, the tangent modulus method determines the actual AE-relation to the stress or strain. The paper provides the mathematical background to the analysis and presents examples of the method. An example is used to show that the method also lends itself well to evaluate the accuracy and reliability of the measurements.